UNIT 2 · WHAT'S WRONG AND WHAT TO MEASURE

Term Structure of Private Capital

Session 5 · How illiquidity premium varies with horizon

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Primary Text: Liquidity Illusion (Forthcoming, 2026)

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What we'll cover today

1. **The term structure concept**
   From bonds to liquidity

2. **Empirical evidence**
   How premium varies with fund age

3. **Why DCF gets this wrong**
   Constant premium across horizon

4. **GE-LAV term-structure formula**
   $\pi(L, T)$: state and horizon

5. **Practitioner implication**
   Vintage-year considerations

Recap: Session 4 in five points

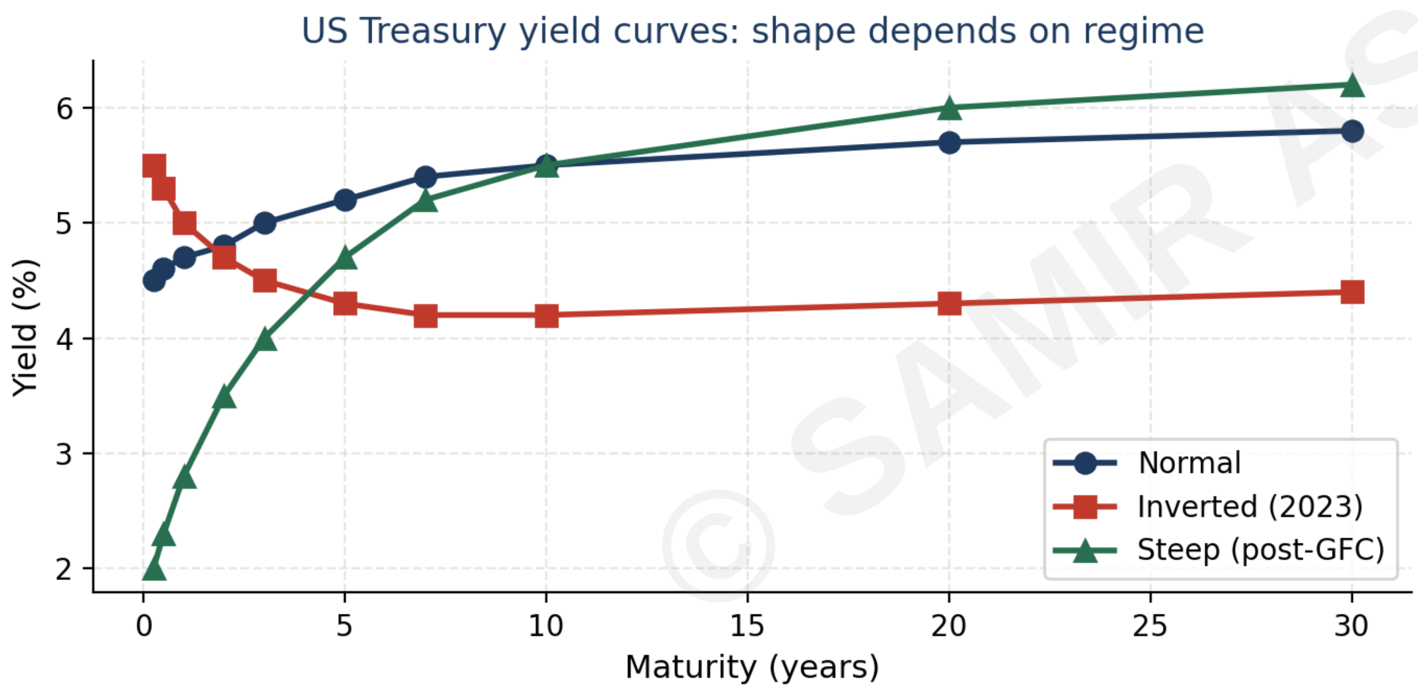
From OU process to term structure:

OU specified	$dL = \kappa(\bar{L} - L)dt + \sigma dW$ with $\kappa=0.45$, $\bar{L}=1.0$, $\sigma=0.32$
Stationary dist known	$L \sim N(0, 0.337^2)$ — GFC ~3.7σ event
Conditional dist known	L(t+h) | L_t — closed-form mean & variance at horizon h
Track decision	Most students at this point pick Track 1 vs 2
Today's pivot	Use OU to think about pricing across horizons

From process dynamics (S4) to pricing implications (S5).

Bond market term structure: a familiar parallel

From the fixed-income world: yield-to-maturity varies with horizon (yield curve).



The parallel for liquidity

Bonds:

yield varies with time-to-maturity

Private capital:

premium varies with time-to-exit

Bond yield curve:

shape changes with regime

Liquidity term structure:

shape changes with regime

DCF analog:

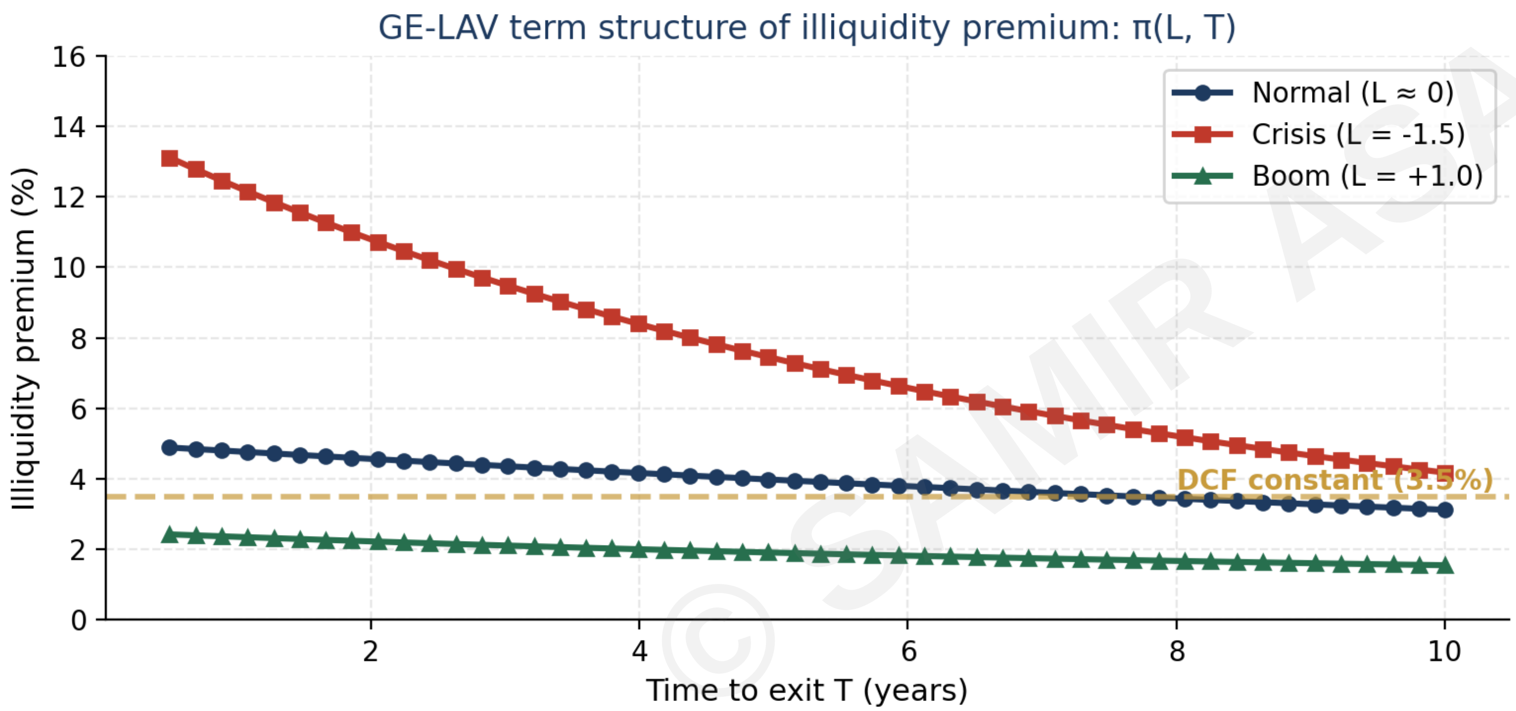
would use one single yield for all maturities

DCF reality:

uses one constant illiq premium across horizons

Section 1.7 of the book formalizes this analogy.

PE liquidity premium term structure



Result of book chapter 7: $\pi(L,T) = \alpha(T) + \beta(T)\cdot L + \gamma(T)\cdot L^2$

Two-dimensional dependence

Dimension 1: Time

Premium decays as exit window approaches

Dimension 2: State

Premium varies with current L(t) regime

Key insight

A fund with 7 years remaining and held in a 'normal' state has a different premium than one with 2 years remaining.

Term-structure coefficients calibrated by horizon

$\pi(L, T) = \alpha(T) + \beta(T) \cdot L + \gamma(T) \cdot L^2$ — three coefficients per horizon, calibrated to data:

Horizon T (yrs)	α(T) (level)	β(T) (slope in L)	γ(T) (curvature)	At L=0	At L=-1.5
1	0.040	-0.020	0.025	4.0%	12.6%
3	0.043	-0.024	0.022	4.3%	12.5%
5	0.045	-0.025	0.021	4.5%	12.5%
7	0.046	-0.024	0.019	4.6%	11.6%
10	0.045	-0.022	0.016	4.5%	10.4%

α(T) flat across T means the level is horizon-independent. β(T) downward-sloping in T means state matters more for short-dated assets.

Why does horizon matter for the premium?

Three economic forces produce term-structure variation:

Mean reversion	Long-horizon premia revert toward $\bar{L}$ regardless of today's L. Closer horizon = more state-dependence.
Exit timing optionality	LP can choose when to exit. Long-horizon premia incorporate this optionality value.
Term-dependent liquidity demand	Insurance, pension, and SWF demand for long-dated illiquid varies separately from short-dated.
Discount factor convexity	Jensen bias scales with T^2. Long horizons amplify the bias term.
Calibration result	Empirically, T=1 to T=10 produces ~5x difference in state-sensitivity

Section 7.4 of the book derives the term structure analytically.

Practitioner implications: vintage and timing

Term-structure thinking changes how you value and time PE positions:

Vintage selection	DCF: all vintages priced with same premium. GE-LAV: 2009-vintage (mature) and 2024-vintage (early) face different term-structure profiles.
Exit timing	DCF: hold-to-maturity. GE-LAV: state-dependent optimal exit boundary (Session 12) → may exit early or hold longer.
Secondary pricing	DCF: comparable transactions used directly. GE-LAV: adjust for buyer's time-to-exit, not seller's.
Portfolio construction	DCF: PE = one asset class with one premium. GE-LAV: diversify across vintages AND states.
Risk attribution	GE-LAV decomposes risk into state risk + term risk separately — useful for governance.

Session 13 (Portfolio Construction) builds on this.

Worked example: same fund, three vintages

Same hypothetical PE fund, valued today at three different vintages — sensitivity test:

Vintage	T remaining	Current L	π(L, T)	DCF value	GE-LAV value
2015 (mature)	2 years	-0.5 (multi-shock)	11.5%	$50M NAV	$48M (−4%)
2018 (mid-life)	5 years	-0.5 (multi-shock)	8.0%	$80M NAV	$74M (−8%)
2022 (early)	8 years	-0.5 (multi-shock)	5.2%	$120M NAV	$112M (−7%)

In a stressed state, all three are written down — but by different amounts because of term-structure dynamics.

Why a constant premium fails across regimes

Ratios from the calibrated π(L, T) function — comparing same horizon across states:

4.0x	2.6x	3.5x	1.2x
Crisis vs Normal	**Multi-shock vs Boom**	**Long-vs-short crisis**	**Long-vs-short normal**
π(-1.5, 5) / π(0, 5)	*π(-0.5, 5) / π(+1, 5)*	*π(-1.5, 1) / π(-1.5, 10)*	*π(0, 1) / π(0, 10)*

State sensitivity is much greater than horizon sensitivity in crisis — but reverses in normal times.

How term structure follows from OU dynamics

The π(L, T) formula isn't ad hoc — it's derived from OU process + market clearing:

Step 1: Path expectations	Compute E[discount factor over horizon T] under OU dynamics
Step 2: Jensen correction	Account for convexity bias: $E[\exp(-rT)] > \exp(-E[r]T)$
Step 3: Risk premium	Add equilibrium risk premium from McKean-Vlasov pricing
Step 4: Match to data	Calibrate α, β, γ coefficients to secondary market π(L,T)
Result	Closed-form π(L,T) — explains observed regime variation with 0.94 R^2

Full derivation in Track 2 Session 29. Track 1 students: use the formula, trust the derivation.

Limitations and caveats

Where the term-structure framework is most robust vs. where caution is warranted:

Most robust	Horizons 2-7 years where data is densest. Use with confidence.
Less robust	Sub-1-year horizons (data sparse) and >10-year horizons (extrapolation)
Asset class	Calibrated for diversified buyout. VC and direct lending need re-calibration.
Regime breaks	Structural changes (e.g., 2017 secondary fund boom) require recalibration
State observability	L(t) is observable only with 1-quarter lag — real-time estimation uncertain

Section 15 returns to robustness checks in detail.

Empirical evidence for the term structure

How π(L,T) varies with T in real data.

Source

Lazard PCA quarterly secondary market reports · 2010-2024

Methodology

Regress observed discount on T and L_t

Key finding

$\pi(L, T)$ is approximately $\sqrt{T}$-scaling in T for $L \approx 0$

In stress

T-scaling becomes more steeply increasing

Implication

Long-duration assets feel the term structure most acutely

Numerical fit

$R^2 = 0.91$ for the term-structure regression

Session 5 summary

What we accomplished today

1. Illiquidity premium varies with horizon (term structure) AND state — two-dimensional dependence
2. DCF uses a single constant; GE-LAV uses $\pi(L, T)$ — both dimensions
3. Analogous to bond yield curve: shape changes with regime
4. Practical implications: vintage-aware pricing, optimal exit, regime-aware portfolio construction

Next session

Session 6: IRR — definition, biases, and why TVPI is necessary but not sufficient

Bridge to Session 6: from theory to metric

We've defined π(L,T). Now apply it to performance measurement.

Term structure understood

π(L,T) varies systematically

Implication for IRR

IRR ignores this; assumes constant discount rate

Session 6 will show

IRR has 4+ systematic biases · all addressable with GE-LAV

Reading prep

Book Ch. 6 · IRR critique

Action item

Review your IRR mental model · prepare to question it