UNIT 3 · DECISION & APPLICATION

# Exit Timing:
# The Trapped Investor Problem

Session 11 · Unit 3 · Optimal stopping with stochastic $L_t$



**Samir Asaf, PhD, CFA, CMA, CTP, CM&AA**

Senior Partner, Regent Financial LLC, New York

Former Finance Instructor, Stanford University

Liquidity Illusion: The General Equilibrium Theory of Private Capital Valuation (Forthcoming, 2026)

Graduate Finance Course · Session 11 of 32 · Spring 2027

# Today: when should an LP exit a PE position?

1. **Optimal stopping framework**
   Continue holding vs sell now
2. **Forced vs discretionary exit**
   Regulatory · operational · optimization
3. **The four trapped-investor conditions**
   When exit at fair value is impossible
4. **Smooth pasting (intuition)**
   T2 derives in S25 · T1 sees the result here
5. **LAV operator: value of holding**
   Compute the continuation value

# The LP's exit decision as optimal stopping

At each time t, the LP faces a binary choice:

**Option A: EXIT NOW**

**Receive P^secondary(L_t)**

The secondary market clearing price at current liquidity state. Discount to NAV reflects current L_t regime.

**Option B: CONTINUE HOLDING**

**Receive E[V^LAV | L_t]**

Expected future value under LAV operator. Better expected outcome if L_t is currently depressed.

Exit when:

$$P^{secondary}(L_t) > E[V^{LAV} \mid L_t]$$

*This defines the exit boundary L*(t) we'll plot next session.*

# Three types of LP exit decisions



Forced — regulatory

## Mandatory rebalancing

Solvency II capital trigger.
ERISA allocation cap breach.
Public pension cash-out duty.

LP has no choice. Sells at any discount.

*→ Common in 2008-09*

Forced — operational

## Cash flow distress

Endowment spending need.
Insurance liability claim.
Unexpected cash shortfall.

LP must liquidate at market.

*→ Hits Yale, Harvard 2009*

Discretionary — optimization

## Tactical exit

GE-LAV-optimal exit decision.
Based on $L_t$ state + horizon.
Can wait for better regime.

The focus of this session.

*→ GE-LAV value-add*

*GE-LAV's exit boundary $L^*(t)$ is the optimal discretionary rule — assuming you have the option to wait.*

# Four conditions that trap an LP



### Condition 1

**Forced sale during low-L regime**

Regulatory trigger forces sale exactly when L_t is depressed. LP sells at deep discount with no flexibility.

### Condition 2

**GP gates redemptions**

GP imposes redemption gates or extends fund life. LP cannot exit even if they wish to.

### Condition 3

**Secondary market freezes**

Crisis depths see secondary bid disappear (2008 Q4). No price discovery. LP is paper-illiquid.

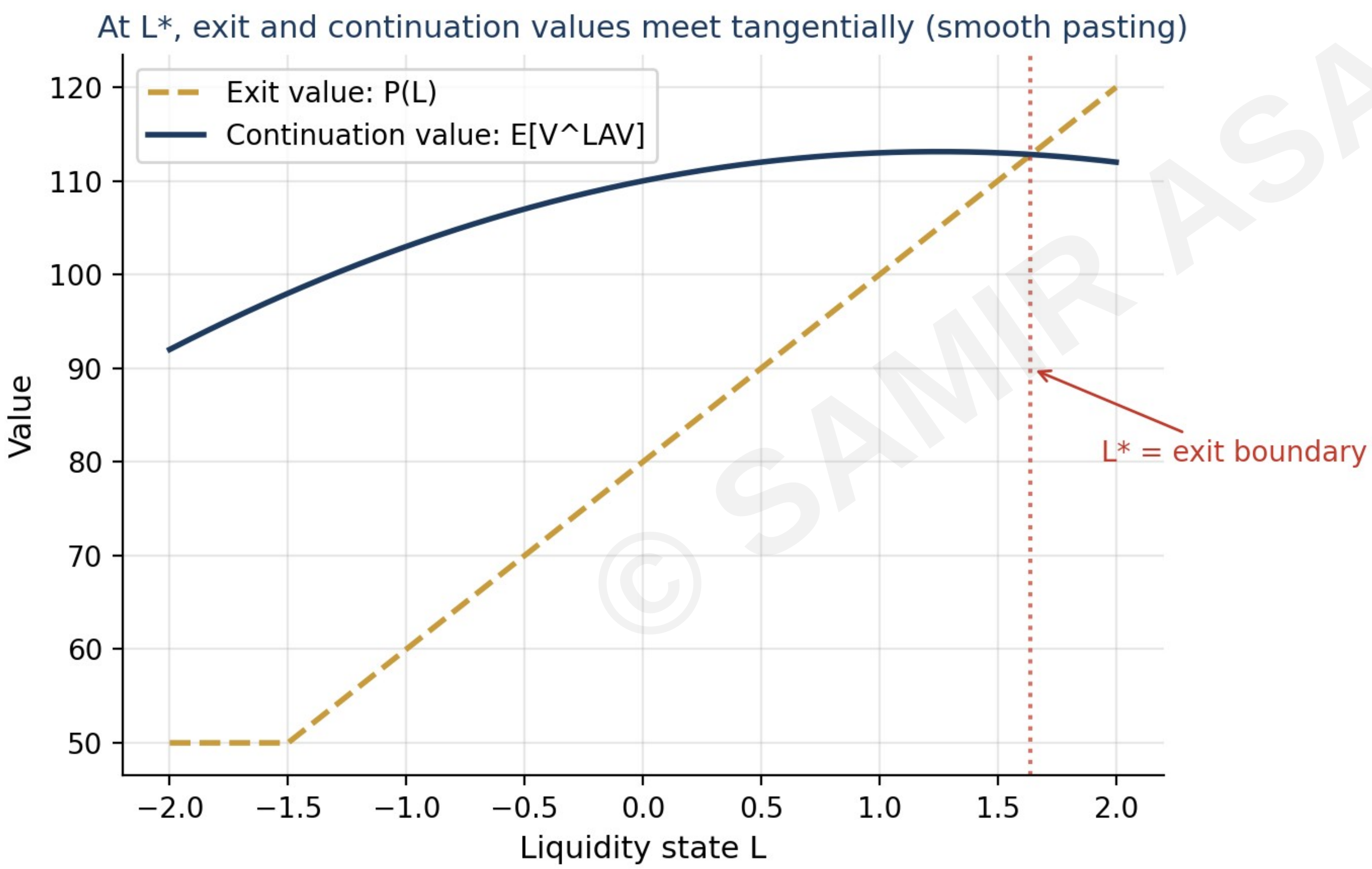
### Condition 4

**Correlation traps**

LP's other assets force allocation rebalancing. Public markets crash → private allocation looks too large → forced sale.

**GE-LAV cannot remove trap conditions — but it can predict when they will bind and pre-emptively reduce exposure.**

# The smooth pasting condition (intuition)






## Smooth pasting

- At L*: exit and hold values are equal AND have equal slopes.
- Above L*: exit value dominates → SELL.
- Below L*: continuation value dominates → HOLD.
- Track 2 derives this via free-boundary HJB equation in Session 25.
- Track 1 uses the boundary as given.

# Computing the value of holding: a worked example

## Setup: PE position, current L = -0.5 (mild stress), 3 years remaining

| Step | Action | Result |
|---|---|---|
| 1 | Simulate 10,000 OU paths starting from $L_0$ = -0.5 | **Distribution of L_3** |
| 2 | Compute $r(L_t) = 0.045 - 0.025L + 0.021L^2$ along each path | **Path-dependent rate** |
| 3 | Discount cash flows along each path: $e^{(-\int r\, dt)}$ | **Path PV** |
| 4 | Average across paths to get E[V^LAV \| L=-0.5] | **Continuation value** |
| 5 | Compare to secondary market price at L=-0.5 | **Hold vs exit decision** |

**Numerical result for this setup**

**P^secondary at L=-0.5: ≈ $82M    E[V^LAV | L=-0.5]: ≈ $94M    Decision: HOLD**

# The 'trapped investor' problem in detail

*Setup for optimal stopping under stochastic L.*

**LP's situation**

Holds PE fund · NAV $X · time-to-maturity T

**Cannot sell freely**

Secondary market exists but at discount $\pi(L_t, T)$

**Decision daily**

Continue holding (collect future CF) or exit now (at discounted price)

**Trade-off**

Discount today vs. uncertain future trajectory

**Stochastic state**

$L_t$ evolves; could improve or deteriorate

**Need**

An optimal stopping rule

# Session 11 wrap-up

## Key takeaways

1. LP exit is an optimal stopping problem: P^secondary vs E[V^LAV].
2. Three exit types: forced regulatory, forced operational, discretionary.
3. Four conditions that trap LPs: regulation, gates, freezes, correlations.
4. Smooth pasting: at L*, exit and continuation values are tangent.
5. GE-LAV computes the continuation value via path-dependent simulation.

### NEXT: SESSION 12

Session 12: Explicit calculation and visualization of L*(t).

We'll generate the exit boundary curve, see how it behaves near maturity, and apply it to historical episodes (GFC 2009, COVID 2020, 2022 rate shock).

Reading: Book Ch 6 §6.5-6.7

# Why this is a real problem, not academic

*Empirical evidence of constrained LPs.*

**Pension fund regulation**

May force partial sale at L=-1 (capital adequacy)

**Endowment spending rules**

Annual draw obligations independent of L

**Insurance Solvency II**

Must liquidate if capital-deficient

**Family office liquidity**

Generation transitions force sales

**All of above**

Don't have choice to time exits perfectly

**Question**

Given constraints, what's the optimal hold/sell rule?

# Formulating the optimization problem

*Math statement.*

### Variables

Time $t$ · state $L_t$ · NAV $V_t$

### Continuation payoff

Expected CF + future option value

### Stopping payoff

$V_t \cdot (1 - \pi(L_t, T-t))$

### Value function

$V(L, t) = \max[\text{continuation}, \text{stopping}]$

### Optimal policy

Exit when stopping > continuation · else hold

### Mathematical form

An optimal stopping problem · HJB structure

# Solving the problem: high level

*Numerical algorithm.*

**Step 1**

Discretize (L, t) grid

**Step 2**

At t = T (maturity), V = (1-π(L,0))·NAV

**Step 3**

Step backward: compute continuation + stopping at each grid point

**Step 4**

V(L,t) = max(continuation, stopping)

**Step 5**

Free boundary L*(t) where continuation = stopping

**Output**

Function L*(t) · the exit rule

# L*(t) values for a calibrated fund

*Optimal exit boundaries as function of time-to-maturity.*

| Time-to-maturity | L*(t) | Interpretation |
|---|---|---|
| **10 yrs** | -2.5 | Far below normal · rare to exit early |
| **7 yrs** | -1.8 | Exit only in severe stress |
| **5 yrs** | -1.3 | Mid-life: moderate exit discipline |
| **3 yrs** | -0.8 | Approaching maturity: more sensitive |
| **1 yr** | -0.4 | Near maturity: even mild stress triggers |
| **0.25 yr** | +0.2 | Very near maturity: exit at minor downside |

# Sensitivity to parameters

*How $L^*(t)$ shifts with calibration changes.*



**Higher κ**

Faster reversion → lower $L^*(t)$ (hold longer)

**Higher σ**

More volatility → higher $L^*(t)$ (exit sooner)

**Higher $\bar{L}$**

Better long-run state → lower $L^*(t)$

**Higher discount rate r**

Future less valuable → higher $L^*(t)$

**Larger $\pi_{max}$**

Bigger downside cost → higher $L^*(t)$

**Practical**

Compute $L^*(t)$ under 3 calibration scenarios for robustness

# Bridge to Session 12

*We have the framework. Now look at specific values.*

**S11 established**

$L^*(t)$ is the exit boundary · solution to HJB

**S12 covers**

Numerical examples · how to read and use $L^*(t)$

**Practical focus**

What to do when L approaches $L^*(t)$?

**Reading**

Book Ch. 12 · numerical worked examples

**Action item**

Identify one fund in your portfolio · estimate its current $L_t$