UNIT 3 · DECISION & APPLICATION

The Exit Boundary L*(t): Numerical Examples

Session 12 · Unit 3 · Reading and applying the optimal exit rule

Samir Asaf, PhD, CFA, CMA, CTP, CM&AA

Senior Partner, Regent Financial LLC, New York

Former Finance Instructor, Stanford University

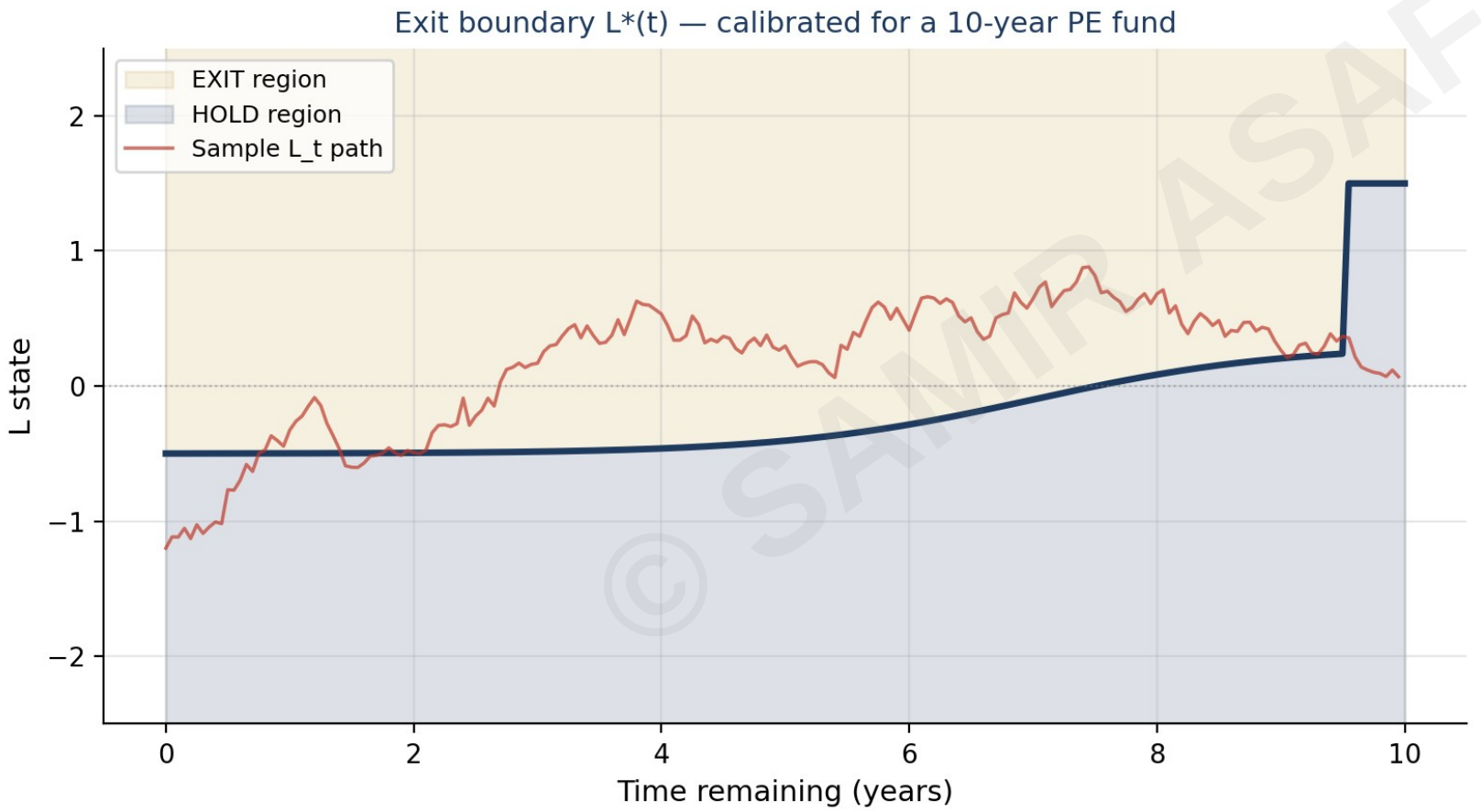
Liquidity Illusion: The General Equilibrium Theory of Private Capital Valuation (Forthcoming, 2026)

Graduate Finance Course · Session 12 of 32 · Spring 2027

Today: from theory to actionable rule

1. **Reading an L*(t) chart**
   Hold region vs exit region
2. **Terminal-time effect**
   Why L*(t) rises as t → T
3. **Compute the exit decision**
   For a given (L, T) pair
4. **Asset-class variation**
   PE vs infra vs private credit
5. **Historical episodes**
   What GE-LAV would have advised

Reading the exit boundary L*(t)



How to read

Gold area

L > L*(t)
EXIT

Navy area

L < L*(t)
HOLD

Red path

Realized L_t.
If crosses curve from below: SELL.

L*(t)→1.5

Near maturity, almost always exit (NAV close to realized).

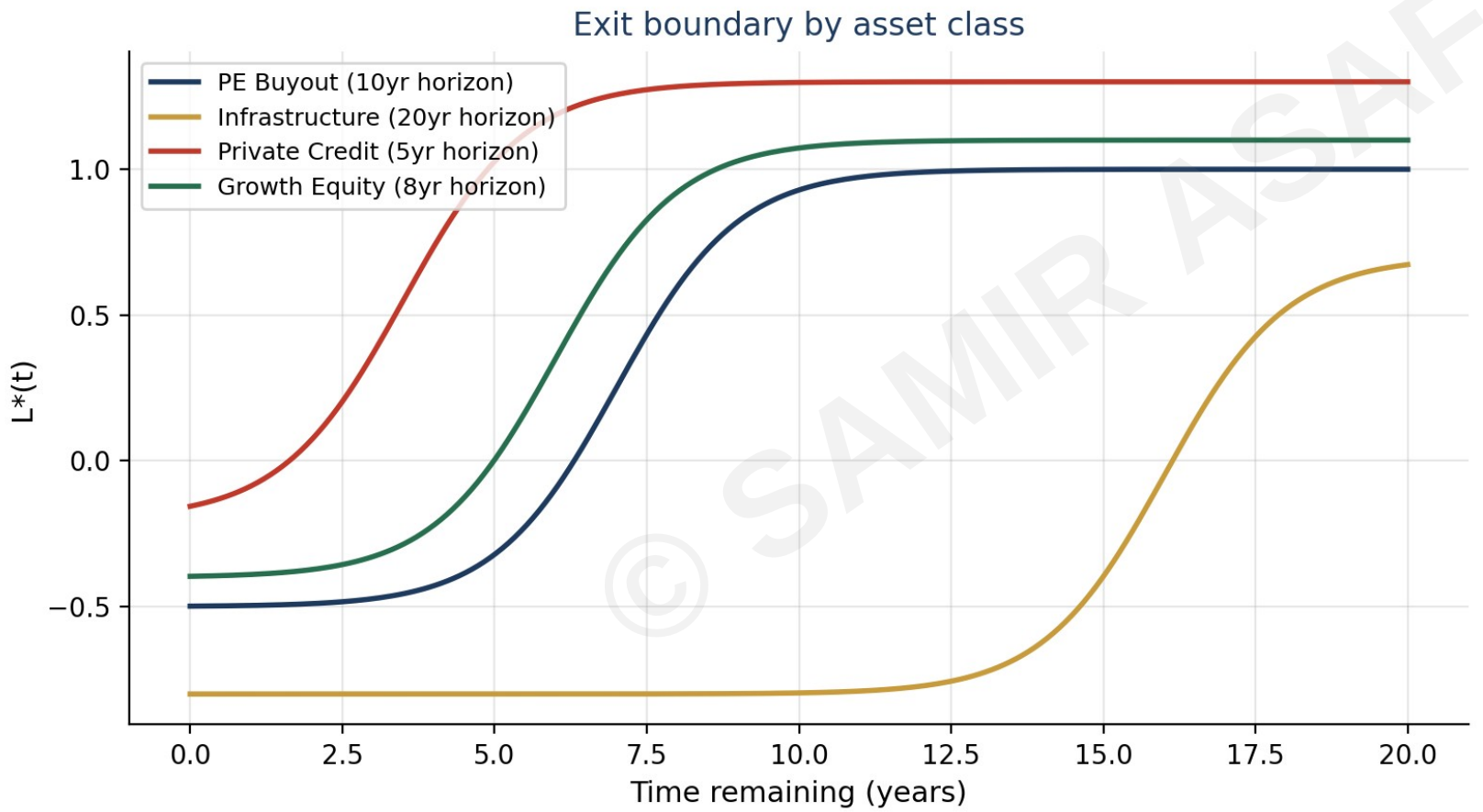
Why L*(t) rises as time runs out

The continuation value depends on remaining 'time to recover.' As T - t shrinks, the value of waiting falls.

Stage	L*(t) range	Intuition
Early life (T-t = 8 years)	**−0.3 to −0.5**	Lots of time for L to mean-revert. Hold even at moderate stress.
Mid life (T-t = 4 years)	**−0.1 to +0.2**	Some recovery time. Exit threshold tightens.
Near maturity (T-t < 1 year)	**+1.0 to +1.5**	Little time left. Sell at NAV (or close to it) — barely any optionality remaining.

American option intuition: optionality value falls as expiry approaches. Same mechanic here.

Exit boundary varies by asset class



Pattern

Short horizon

PC (5yr): boundary close to 0
Little time, narrow flexibility

Medium horizon

PE (10yr): meaningful patience
Wide hold region in early years

Long horizon

Infra (20yr): extreme patience
Hold even at L = -1 for 15yrs

What GE-LAV would have advised: three episodes

Episode	Date	L_t observed	L*(t) for 8yr-remaining PE	GE-LAV advice
GFC trough	Q1 2009	−1.8	−0.45	**HOLD (L far below L*)**
Mid-recovery	2011	+0.2	−0.20	**EXIT marginal funds (L above L*)**
COVID shock	Q2 2020	−0.7	−0.40	**HOLD (L below L*)**
2022 rate shock	Q4 2022	−0.5	−0.30	**HOLD (L below L*)**
Late-cycle PE	Q1 2025	+0.4	−0.10	**EXIT older funds (L above L*)**

Key insight: GE-LAV advises HOLD during stress (counter to forced-sale behavior of stressed allocators).

The optimal rule diverges from what LPs actually did in 2008-09 — and that gap is part of the welfare loss.

Recap: what's L*(t)?

The exit boundary is the heart of LP decision-making under GE-LAV.

Definition

L*(t) = state below which holding is suboptimal; exit immediately

Source

Solution to HJB equation (Session 25) with smooth-pasting BC

Shape

U-shaped over fund life; deeper near maturity

Why it matters

Practical exit rule: observe L_t, compare to L*(t)

Numerical methods

Crank-Nicolson FD scheme on (L, t) grid

Today's focus

How to read and use L*(t) in real decisions

Session 12 wrap-up

Key takeaways

1. $L^*(t)$ plotted as a curve: above = exit, below = hold.
2. Terminal-time effect: $L^*(t) \to +\infty$ as $t \to T$ (no time to wait for recovery).
3. Exit rule varies by asset class: shorter horizons → tighter boundary.
4. Historical episodes: GE-LAV would have advised HOLD during GFC, COVID, 2022.
5. Optimal rule diverges from observed forced-seller behavior in crises.

NEXT: SESSION 13

Session 13: Liquidity traps, GE-LAV historical performance, GP roll/extension decisions.

When the GE-LAV exit rule fails or underperforms. Connection to GP extension decisions. Preview of the Pigouvian tax (full treatment in S24).

Reading: Book Ch 6 §6.7-6.10

$L^*(t)$ values for a calibrated fund

Example fund: Year-10 maturity, OU calibration (κ=0.45, σ=0.32).

t (yrs to maturity)	$L^*(t)$	Interpretation
10 yrs	L^*=-2.5	Almost never optimal to exit early at fund start
7 yrs	L^*=-1.8	Exit if L drops below -1.8
5 yrs	L^*=-1.3	Mid-life: tighter discipline
3 yrs	L^*=-0.8	Late stage: more sensitive to L
1 yr	L^*=-0.4	Near maturity: small dips trigger exit
0.25 yr	L^*=+0.2	Very near maturity: exit even in mild boom

Pattern: $L^*(t) \to \bar{L}$ as $t \to T$ (approaches the long-run mean)

Reading L*(t) in practice

How an LP actually uses this rule.

Step 1

At each quarter end, estimate current L_t (book Ch. 5 methodology)

Step 2

Look up L*(t) for current time-to-maturity (precomputed table or chart)

Step 3

Compare: if L_t < L*(t), exit; else continue

Step 4

Apply hedges between rebalances if L approaching L*

Step 5

Document the decision rationale (regulators need this)

Tool

Excel template available in book companion materials

When L*(t) gives a counter-intuitive answer

Cases where the rule disagrees with practitioner instincts.

Case 1: late-stage normal

L=0, t=1yr from maturity → L*(t)=+0.2 → EXIT

Intuition

Why? Time decay means even minor L volatility now matters

Case 2: mid-life crisis

L=-1.0, t=5yr → L*(t)=-1.3 → HOLD

Intuition

Why? Mean reversion will recover before maturity

Case 3: GFC depths

L=-1.5, t=3yr → L*(t)=-0.8 → EXIT immediately

Intuition

Why? Even with κ recovery, won't reach $\bar{L}$ in time

Sensitivity of L*(t) to parameters

How L(t) shifts with calibration uncertainty.*

Increase κ

Faster reversion → L*(t) lowers (hold longer)

Increase σ

More volatility → L*(t) raises (exit sooner)

Increase $\bar{L}$

Better long-run state → L*(t) lowers (hold longer)

Increase r (discount)

Future less valuable → L*(t) raises (exit sooner)

Increase π_max

Bigger downside → L*(t) raises (be cautious)

Practical advice

Compute L*(t) under 3 calibration scenarios; report the range

Worked example: 2007 vintage fund decision history

Year-by-year L_t and L(t) for a real 2007 vintage.*

Year-end	L_t (realized)	L*(t)	GE-LAV says	Actually did
2008	-1.0	-1.5	Hold	Held
2009	-0.6	-1.3	Hold	Held
2010	0.0	-1.0	Hold	Held
2011	+0.4	-0.7	Hold	Held
2012	+0.5	-0.4	Hold	Sold (secondary 50%)
2013	+0.6	0.0	Hold	Sold remainder
Implication	GE-LAV says LP exited too early in 2012-13	—	—	—

Boundary cases and edge phenomena

Numerical pathologies and how to handle them.

L far below L*

Always exit; don't double-down (no martingale property)

L close to L*

Sensitivity to estimation noise; use buffer (±0.1)

Discrete liquidity events

Capital calls / distributions can shift effective L

End-of-life

Final 6 months: $L^*(t)$ may exceed $\bar{L}$; that's correct

Empty grid points

Some (L, t) combinations have no data; interpolate carefully

Cross-validation

Backtest on prior vintages; report MAE

Bridge to Session 13

We've solved one LP's problem. What about a portfolio of LPs?

Single-fund decision

$L^*(t)$ gives the exit rule for one fund

Portfolio question

Does $L^*(t)$ change when the LP holds multiple funds?

Hedge demand

If funds are correlated, the LP gets hedging benefits

Merton's framework

Portfolio choice with stochastic π is Merton-style

Session 13 will solve

The full LP portfolio problem with hedge demand

Preview

Optimal weights depend on correlation structure of L_t across funds