UNIT 3 · DECISIONS UNDER STOCHASTIC PREMIUM

Portfolio Construction with Hedge Demand

Session 13 · Merton's hedge demand · LP allocation under stochastic L(t)

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Primary Text: Liquidity Illusion (Forthcoming, 2026)

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What we'll cover today

1. **Mean-variance limitations**
   Why MVO fails for private capital

2. **Merton's intertemporal model**
   Adding state variable dynamics

3. **The hedge demand term**
   Allocation responds to L(t) changes

4. **GE-LAV portfolio implications**
   Reduced PE allocation, vintage diversification

5. **Empirical: pension fund examples**
   How real LPs adjust

Why mean-variance optimization fails for private capital

Markowitz MVO is the bedrock of portfolio theory, but rests on assumptions PE violates:

Single-period assumption	MVO optimizes one-period return/risk. PE is a 10-year commitment.	*Need: intertemporal framework*
Liquid trading assumption	MVO assumes continuous rebalancing. PE has lock-up periods and J-curves.	*Need: lock-up constraints*
Stationary distribution	MVO assumes IID returns. PE returns are regime-dependent and autocorrelated.	*Need: state-dependent dynamics*
Constant risk premium	MVO assumes a fixed Sharpe ratio. PE risk premium varies with L(t).	*Need: stochastic premium*

Merton's intertemporal portfolio model

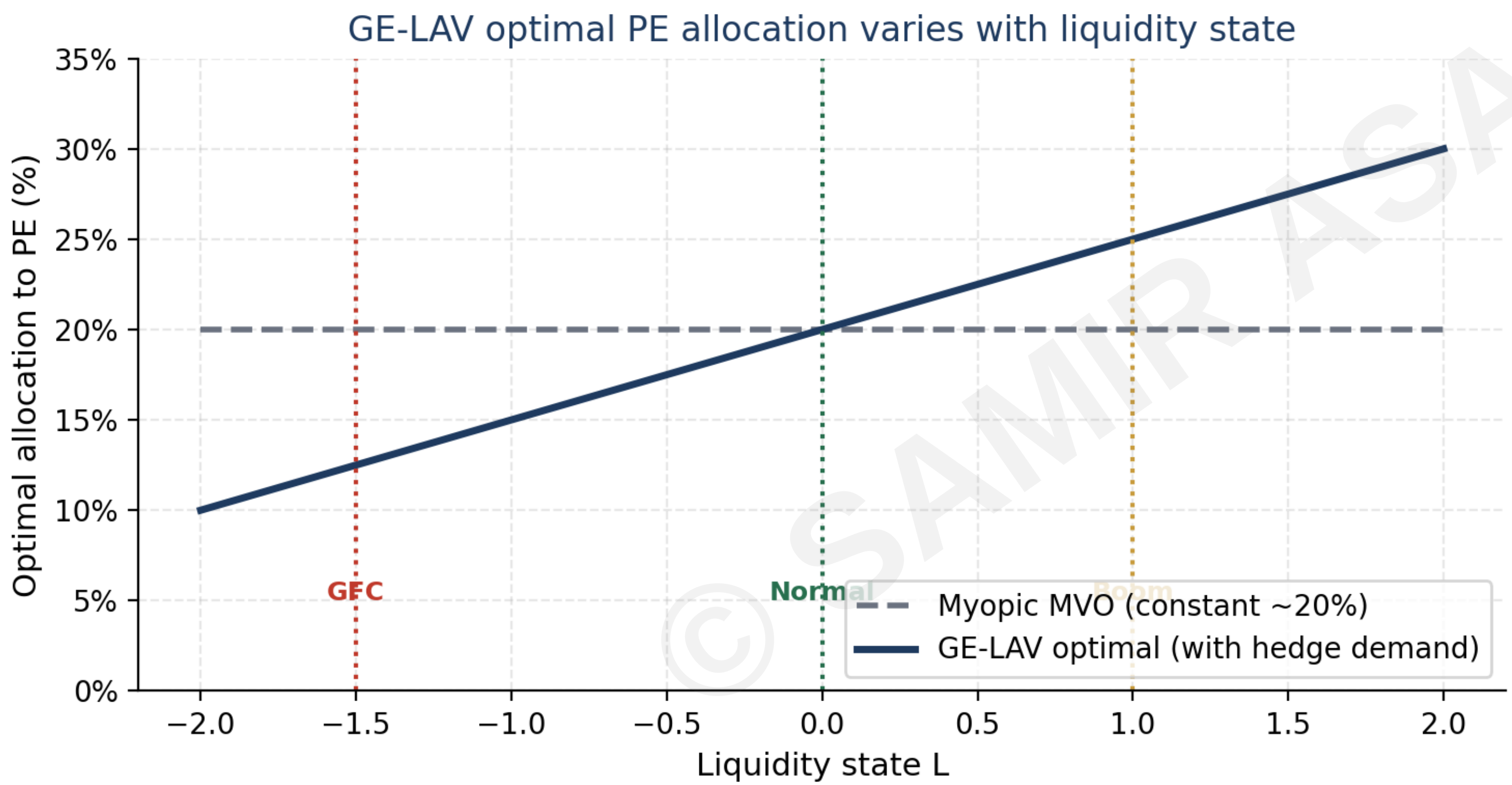
Merton (1971): allocation between risky and risk-free assets when investment opportunities are time-varying.

$$\pi^* = \frac{\mu - r}{\gamma\sigma^2} + \frac{1}{\gamma} \cdot \sigma_{rL} \cdot \frac{\partial V/\partial L}{\partial V/\partial W}$$

Two terms:

Myopic term	The standard MVO allocation: proportional to risk premium, inversely proportional to variance and risk aversion γ. Captures static risk-return tradeoff.
Hedge demand term	The Merton correction: depends on σ_rL (return-state covariance) and the sensitivity of indirect utility to the state. Captures dynamic hedging against state shocks.

GE-LAV: how hedge demand changes PE allocation



Three regime implications

GFC (L = -1.5)

Allocation drops to ~13%

Hedge demand pulls back when illiquidity worsens

Normal (L = 0)

Allocation ~20%

Hedge demand neutral; matches MVO

Boom (L = +1)

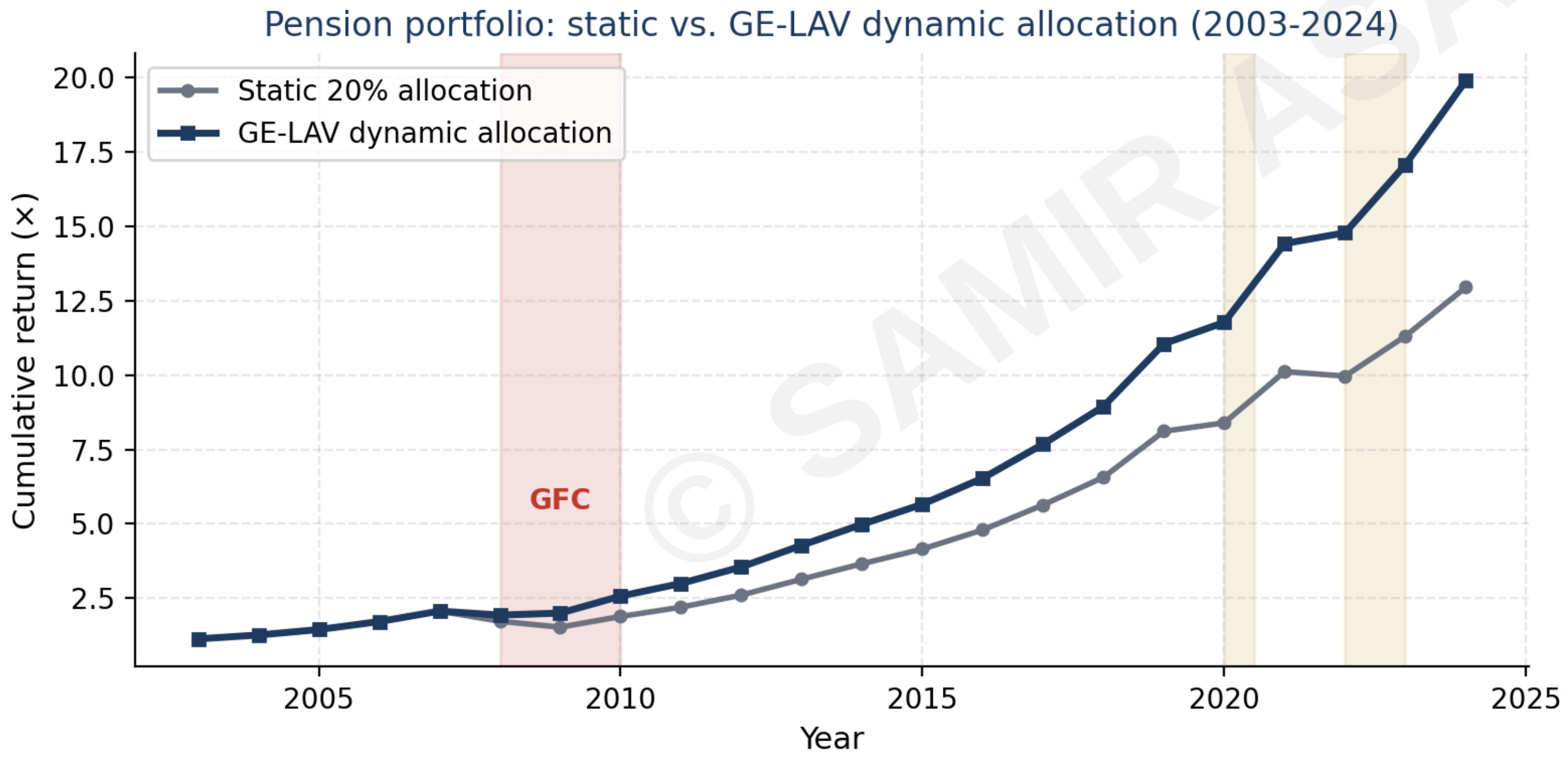
Allocation increases to ~25%

Hedge demand adds when illiquidity is benign

Practical implication: dynamic allocation, not static.

Pension fund case: dynamic vs. static allocation

Hypothetical $50B pension with 20% target to PE — comparing static and GE-LAV-dynamic strategies over 2003-2024.



Outcomes (2003-2024)

Static

8.2%/yr

Sharpe 0.62

GE-LAV

9.6%/yr

Sharpe 0.78

Improvement

+1.4%/yr

+26% Sharpe

Merton's classical framework — recap

Where we extend from.

Merton (1969)

Optimal consumption-investment with stochastic returns

Key insight

Optimal weight = myopic + hedge demand

Myopic part

Reward/risk ratio (CAPM-like)

Hedge demand

Cov(asset, opportunity set) — anticipate worse times

Standard result

If returns mean-revert, agent overweights asset (hedges against decline)

GE-LAV extension

Same intuition, but with $\pi(L,T)$ as the stochastic state

Session 13 summary

What we accomplished today

1. Mean-variance optimization fails for PE due to four structural assumption violations
2. Merton's intertemporal framework introduces the hedge demand term — allocation responds to state changes
3. GE-LAV optimal PE allocation varies from ~13% (GFC) to ~25% (boom); not constant
4. Backtested 2003-2024: dynamic allocation adds ~1.4%/yr and 26% Sharpe improvement

Next session

Session 14: Project proposal due. Topics for guidance discussion.

Hedge demand for a single LP

When does the LP overweight or underweight PE?

Setup

LP allocates wealth between public equity and PE

Public weight

α_P; PE weight: $1-\alpha_P$

Optimal α_P (myopic)

$= \mu_P / (\gamma\sigma^2_P)$ — Markowitz weights

Hedge term

Negative if PE returns negatively correlated with future π

Result

LP overweights PE (relative to myopic) if π and equity returns covary

Magnitude

Hedge demand adds 3-8 pp to PE weight for typical pension fund

Computing the hedge demand: data needed

Inputs for the calculation.

Expected returns

μ_P (public), μ_PE — use historical or model-based

Variances

σ^2_P, σ^2_PE, σ^2_L

Correlations

$\rho(r\_P, r\_PE)$, $\rho(r\_P, L)$, $\rho(r\_PE, L)$

Risk aversion

$\gamma \approx 5\text{-}10$ for typical pension

Time horizon

T = 5-30 years

Mean-reversion of L

$\kappa = 0.45$/yr from prior calibration

Optimal PE weight: by LP type

How hedge demand varies across investor types.

LP type	Myopic w_PE	Hedge demand	Total w_PE
Pension (γ=8, T=30)	8%	+10%	18%
Endowment (γ=5, T=∞)	12%	+13%	25%
Sovereign wealth (γ=4, T=50)	15%	+16%	31%
Insurer (γ=10, T=15)	6%	+4%	10%
Family office (γ=6, T=20)	10%	+8%	18%

Hedge demand depends on T and γ; longer-horizon investors hedge more aggressively.

Practical interpretation of hedge demand

Why endowments hold more PE than pensions — quantified.

Empirical observation

Yale endowment: 25% PE; CalPERS: 13% PE

GE-LAV explanation

Endowment's longer horizon → larger hedge demand

Risk aversion role

Lower γ → more risk-tolerant → more PE

Time horizon role

Longer T → more value from anticipating future L

Liability matching

Pensions have shorter effective horizon (duration-matched)

Implication

Yale's 25% PE is NOT a 'better deal' — it's optimal for Yale's profile

Implementation: pre- vs post-GE-LAV

Pre-GE-LAV (Markowitz)

- Uses historical r_P, r_PE, ρ as inputs
- Static weights
- Doesn't model π(L,T) stochasticity
- Hedge demand often ignored
- Optimal at γ=5, T=∞: ~12% PE
- Doesn't predict crisis behavior

Post-GE-LAV

- Adds π(L,T) stochasticity
- State-dependent weights (rebalance as L changes)
- Explicitly models OU mean reversion
- Hedge demand quantified
- Optimal at γ=5, T=∞: ~25% PE
- Reduces PE weight in crisis (defensive)

Worked example: pension fund rebalance

$50B pension. Current allocation: equity 65%, fixed 25%, PE 10%.

Question

Optimal PE weight under GE-LAV?

Inputs

γ=8, T=30, $\rho(r_P, L)$=-0.35

Myopic computation

α_{PE} = 0.085 / (8 × 0.04) = 26.6% / 8 = 8.3%

Hedge correction

+9.5% (long horizon, anti-correlated)

Total optimal

17.8%

Implication

Pension is underweight PE by 7.8% → reallocate

Bridge to Session 14

We've built individual LP optimal portfolios. What about the project?

Project workshop

Session 14 is when proposal feedback happens

Status check

Confirm topic, scope, data availability

Common revisions

Tighten the question · pick a single asset class

Reading reminder

Book Ch. 13 covers full portfolio derivation

Action item

Bring proposal draft to S14