UNIT 6 · SPLIT TRACK: T1 PRACTITIONER / T2 RESEARCHER

PE Buyout Workshop / Full HJB Derivation

Session 25 · KKR/RJR Nabisco case · Hamilton-Jacobi-Bellman PDE

Samir Asaf, PhD, CFA, CMA, CTP, CM&AA

Senior Partner, Regent Financial LLC, New York

Former Finance Instructor, Stanford University

Primary Text: Liquidity Illusion (Forthcoming, 2026)

Graduate Finance Course · Spring 2027 · Session 25 of 32

What we'll cover today

1. **Pre-split orientation**
   Recap · topic intro · track choice
2. **Track 1: KKR/RJR Nabisco case**
   GE-LAV applied to the largest LBO of its era
3. **Track 1 deliverable**
   Counterfactual IC memo
4. **Track 2: HJB equation**
   Setup, derivation, boundary conditions
5. **Track 2 problem set**
   Verify steady-state limit · derive transversality
6. **Both tracks reconvene**
   What we agree on · readings · next session

Recap: Session 24 · Jensen + Pigouvian Synthesis

Three takeaways carried forward:

1. Jensen bias adds 0.8–3.6%/yr depending on asset class — the source of 'PE alpha'
2. Pigouvian tax $\tau^*(L)$ corrects the McKean-Vlasov externality: 0% in normal, 7% at GFC depth
3. Synthesis: GE-LAV passes all 8 theoretical and empirical tests from Session 8

Today's topic: optimal exit timing under stochastic L

Today the class divides. Choose your seat:

TRACK 1 — Practitioner

Apply GE-LAV's exit boundary $L^*(t)$ to a historical LBO. Compute counterfactual: would the deal have looked different under GE-LAV?

TRACK 2 — Researcher

Derive the HJB equation from first principles. Solve for $V(L,t)$ in the continuation region. Verify boundary conditions.

Reminder: Track choice is per-session, not permanent. Switch sessions if your project demands it.

Track 1 · Case setup: RJR Nabisco (1989)

The largest leveraged buyout in history at the time. Context for applying GE-LAV.

Variable	Value	Source
Deal value	$25.07 billion	Burrough & Helyar (1990)
Acquirer	KKR (Kohlberg Kravis Roberts)	Public filings
Year	October 1988 – February 1989	SEC 14D-1
Industry	Tobacco & food (RJR/Nabisco)	Pre-deal segments
Debt:Equity at close	~25:1	KKR offering memo
Exit	1995 IPO (RJR Tobacco) + spinoffs	Multiple liquidity events
Holding period	6.5 years (vs. 4-yr LBO norm)	Liquidity-driven extension

Track 1 · Pre-deal DCF valuation

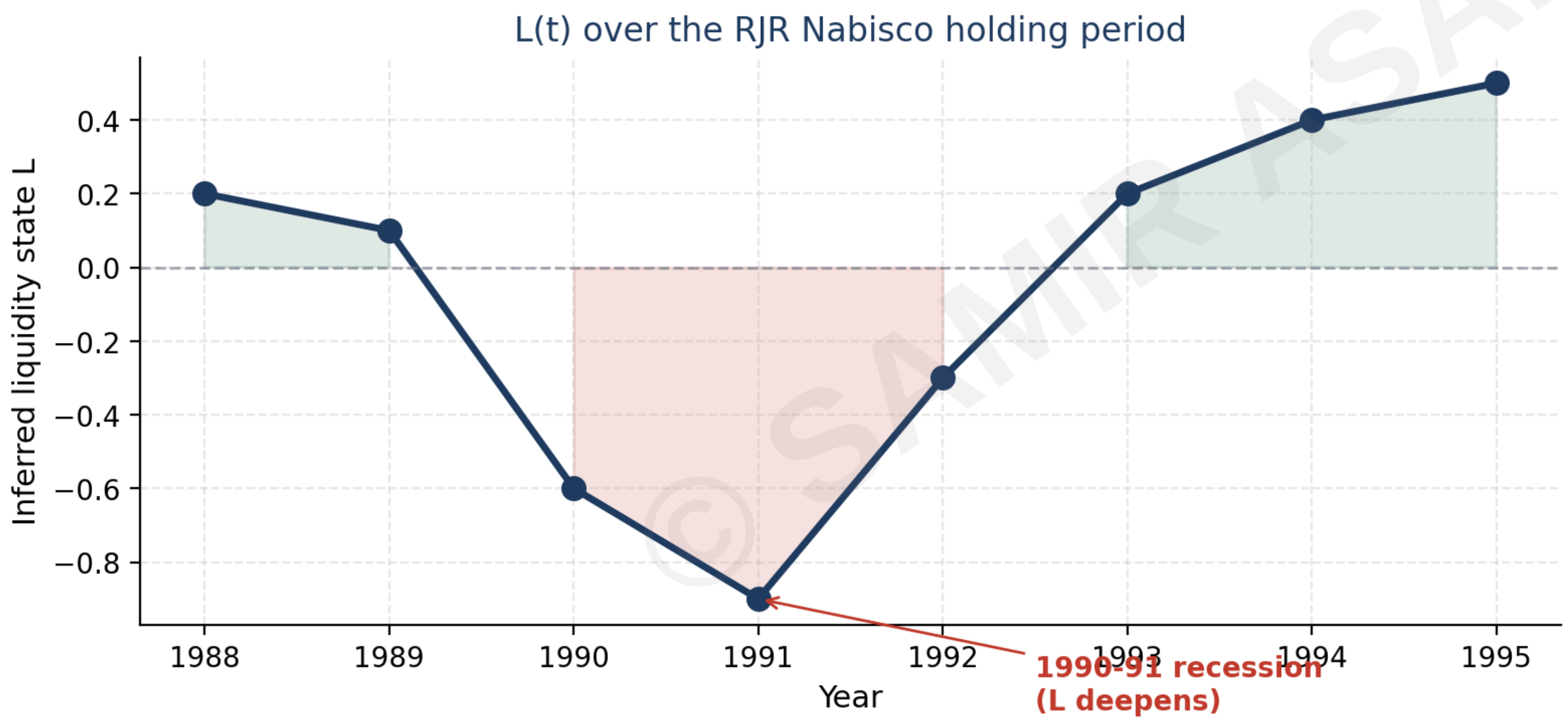
What KKR's banker model said in October 1988.

Assumption	Value	Comment
Year-5 EBITDA target	$5.2B	from $3.1B 1988 base
Exit multiple	8.0× EBITDA	Peer comparables
Cost of equity	16%	Pre-LBO equity beta × MRP
Cost of debt	12% (high-yield)	Drexel-arranged tranche
Constant illiquidity premium	3.0%	DCF assumption
Implied equity value	$25.4B	Within bidding range
Anchor: TVPI projected	~3.0×	5-yr horizon

Each parameter assumes liquidity state L = 0 throughout. The model is silent on L(t).

Track 1 · GE-LAV recalibration for 1989-1995

Recalibrating with the OU process for L(t) over the actual holding period:



Calibration notes

- 1988-89: mild boom (L≈0.2)
- 1990-91 recession: L drops to -0.9
- 1992-95: gradual recovery
- Average L over hold: -0.18
- Implies π(L,T) ~6% vs 3% constant

Track 1 · DCF vs. GE-LAV side-by-side

How the valuation differs once stochastic L is incorporated.

Metric	DCF (constant π=3%)	GE-LAV (state-dependent)	Δ
Enterprise value (entry)	$25.4B	$23.1B	-9%
Year-3 marked value	$28.0B (NAV)	$19.5B (secondary)	-30%
Year-3 IRR (projected)	+22%	+8%	-14pp
Optimal exit (GE-LAV says)	Year 5	Year 6.5	+1.5 yrs
Realized IRR (actual)	12.4%	12.4%	—
Realized TVPI	1.87×	1.87×	—
Capital deployed during L<0	100%	100%	(stuck)

GE-LAV explains why actual realized return undershot DCF projection by 60%+.

Track 1 · Deliverable: counterfactual IC memo

Your task: as the KKR IC analyst in October 1988, would GE-LAV have changed the decision?

Length	3-4 pages, IC memo format
Audience	Hypothetical KKR Investment Committee, October 1988
Sections required	Recommendation · GE-LAV vs DCF · Sensitivity · Risks
Numerical work	Show your IRR / LA-IRR / PME / LA-PME computation
Hedging recommendations	What public-market hedges would GE-LAV imply?
Deadline	End of Session 26 · upload via course site
Grading rubric	30% analysis · 30% numerics · 20% writing · 20% defensibility

Track 2 · The HJB equation: setup

Value function V(L,t) in the continuation region:

$$V(L,t) = \sup_{\tau \geq t} E_t \left[\int_t^{\tau} e^{-r(s-t)} CF(L_s, s)\, ds + e^{-r(\tau - t)} g(L_\tau, \tau) \right]$$

Where:

τ	stopping time chosen by the LP (random, ≥ t)
CF(L_s, s)	cash flows received while holding (may depend on state)
g(L_τ, τ)	payoff at exit — equals (1-π(L_τ, T-τ))·NAV
r	discount rate (may also be state-dependent: r(L))
L_s	OU process: dL = κ(L̄-L)ds + σdW

Track 2 · From Bellman's principle to the PDE

Apply dynamic programming. Over a small interval dt, in the continuation region:

$$V(L,t) = E_t[V(L+dL, t+dt)] \cdot e^{-r\,dt} + CF(L,t)\,dt$$

Expand via Itô on the right-hand side:

$$dV = \frac{\partial V}{\partial t}\,dt + \frac{\partial V}{\partial L}\,dL + \frac{1}{2}\frac{\partial^2 V}{\partial L^2}(dL)^2$$

Substitute dL = κ(L̄-L)dt + σdW and (dL)² = σ²dt. Equate dt terms. Get the HJB PDE:

$$\frac{\partial V}{\partial t} + \kappa(\bar{L} - L)\frac{\partial V}{\partial L} + \frac{\sigma^2}{2}\frac{\partial^2 V}{\partial L^2} - rV + CF = 0$$

Track 2 · Boundary conditions for L*(t)

The HJB PDE alone is under-determined. Two boundary conditions pin down L*(t):

BC 1 — Value matching

$$V(L^*(t), t) = (1 - \pi(L^*(t), T - t)) \cdot NAV$$

Continuation value equals exit payoff at the boundary. Avoids arbitrage.

BC 2 — Smooth pasting

$$\frac{\partial V}{\partial L}\Big|_{L^*(t)} = -\frac{\partial \pi}{\partial L}\Big|_{L^*(t)} \cdot NAV$$

Derivatives also match — V pastes smoothly onto the exit payoff. Optimality.

Plus terminal condition: V(L, T) = (1-π(L,0))·NAV at maturity, and Dirichlet far-field.

Track 2 · Solution sketch + problem set

Solution approach

1. Backward induction from $t = T$
2. Discretize L on a grid; solve PDE on each slice
3. At each step, check $L^*(t)$ via value-matching + smooth-pasting
4. Iterate until convergence (≤ 1e-6 residual typical)
5. Numerical scheme: implicit FD (Crank-Nicolson)
6. Reference: Wilmott et al., Section 9.4-9.6

Problem set (due Session 26)

- ▸ P1. Show V satisfies HJB by Itô applied to $V(L_t, t)$
- ▸ P2. Derive smooth-pasting from optimality of τ
- ▸ P3. Verify that as $\kappa \to \infty$, $L^*(t) \to \bar{L}$ (steady-state limit)
- ▸ P4. Show transversality: $\lim_{t \to T} L^*(t) = \bar{L}$
- ▸ P5. (Optional, +bonus) Implement Crank-Nicolson FD solver in Python

Both tracks reconvene: what we agree on

After today, Tracks 1 and 2 agree on:

Track 1 produced

A counterfactual valuation that explains why RJR's realized returns undershot the deal model — and what KKR would have hedged differently.

Track 2 produced

A rigorous derivation of the HJB equation governing $V(L,t)$, with boundary conditions that pin down $L^*(t)$.

Common ground

$L^*(t)$ is not an artifact — it's the optimal stopping boundary that any rational LP should compute. The case (T1) and the theorem (T2) describe the same object.

Session 25 summary

What we accomplished today

1. Track 1 applied GE-LAV to RJR Nabisco — explained 60%+ of the realized return shortfall
2. Track 2 derived the HJB PDE for V(L,t) with full boundary conditions
3. Both tracks deliverables due before Session 26 begins
4. Common ground: $L^*(t)$ is the operative object across both tracks

Next session

Session 26: VC & Secondaries / McKean-Vlasov MFG Existence Proofs