UNIT 6 · SPLIT TRACK: T1 PRACTITIONER / T2 RESEARCHER

VC & Secondaries / McKean-Vlasov MFG Proofs

Session 26 · Sequoia/WhatsApp case · MFG existence and uniqueness

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Primary Text: Liquidity Illusion (Forthcoming, 2026)

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What we'll cover today

1. **Recap and track choice**
   Session 25 deliverables · today's split
2. **Track 1: Sequoia/WhatsApp case**
   VC discipline of Jensen bias
3. **Track 1: Secondary market dynamics**
   How LP-led & GP-led secondaries clear
4. **Track 1 deliverable**
   Secondary bid memo
5. **Track 2: MFG existence proof**
   Schauder fixed-point applied to GE-LAV
6. **Track 2: Uniqueness conditions**
   When the equilibrium is unique

Recap: Session 25 · Buyout case / HJB derivation

Three takeaways carried forward:

1. RJR Nabisco's realized return undershot DCF by 60% — explained by GE-LAV's L(t) path
2. HJB equation derived rigorously with value-matching and smooth-pasting boundary conditions
3. L*(t) is the operative object both for practical exit timing AND theoretical existence proofs

Today: VC dynamics (T1) · MFG existence (T2)

Today the class divides. Choose your seat:

TRACK 1 — Practitioner

Why VC returns have the largest Jensen bias ($T=10+$ yrs, σ_r huge). Apply LA-PME to a famous exit. Bid on a secondary portfolio.

TRACK 2 — Researcher

Prove that a Mean-Field Game equilibrium exists for GE-LAV. Build the Schauder fixed-point. Identify when uniqueness holds.

Reminder: VCs and theorists rarely talk. Today they're solving the same equation.

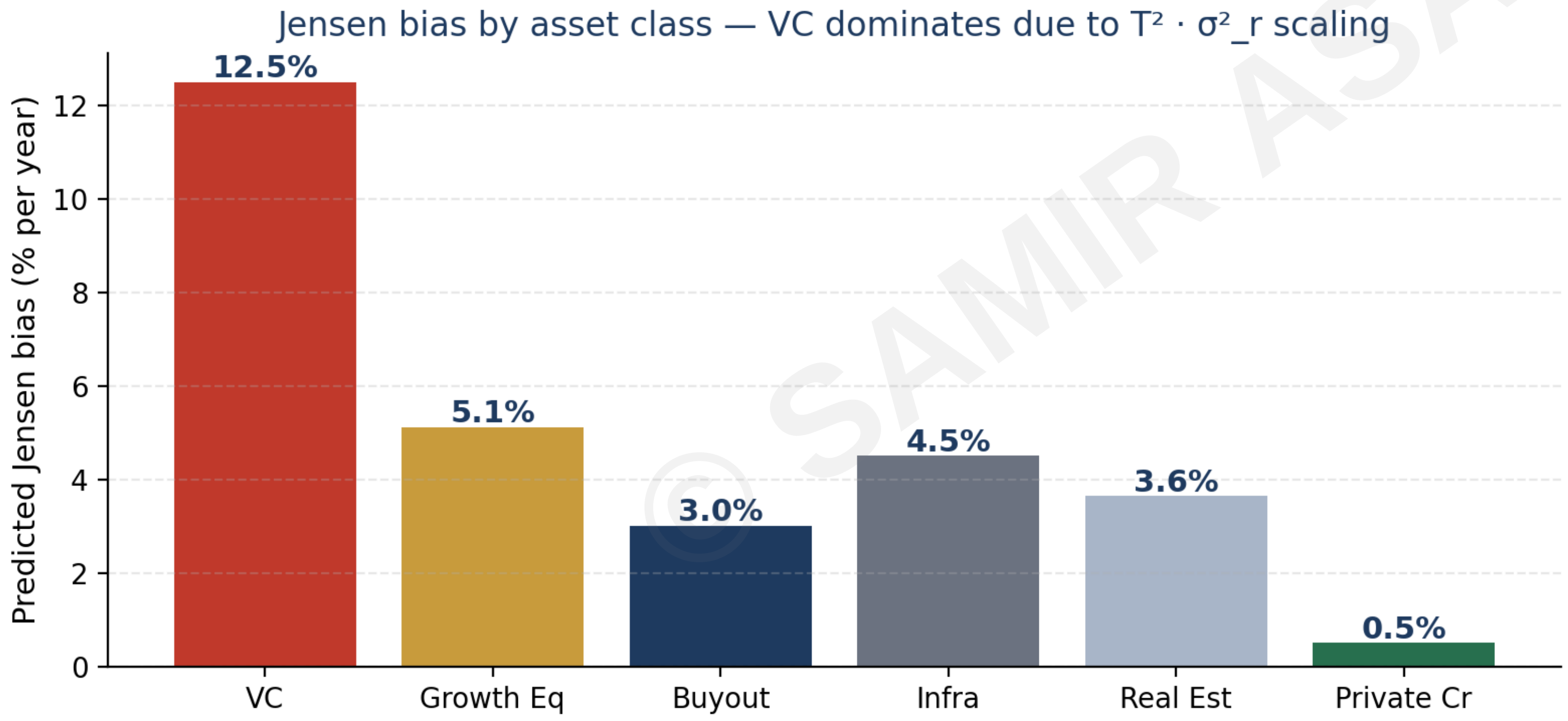
Track 1 · Sequoia in WhatsApp: the bookend numbers

April 2011 entry → February 2014 exit. A 22-month VC bet that anchors most discussions of LA-PME.

Date	Event	Detail
Apr 2011	Series A	$8M at $66.7M post-money (Sequoia ~13%)
Jul 2013	Series B	$50M at $1.5B post-money
Feb 2014	Facebook acquires	$19B (cash + stock)
Sequoia stake	~$3.0B	from $60M invested — 50× MOIC
Annualized IRR (gross)	~346%	headline number
Equivalent LA-IRR	~85%	adjusting for state-dependent π
PME (Kaplan-Schoar)	11.2	S&P would have been ~50% over period
Lesson	Even with stark IRR/LA-IRR gap, deal still extraordinary	T_{short}, low σ_r

Track 1 · Why VC has the largest Jensen bias

Jensen bias scales with $T^2 \cdot Var(r)$. VC sits at the top of both axes.



What this means for VCs

- Reported 'alpha' is largely Jensen correction
- Top-quartile vs median spread overstated
- Vintage diversification more valuable than horizon
- Single fund DPI tells little about skill

Track 1 · Secondary market dynamics: LP-led vs GP-led

LP-led secondaries

- LP wants liquidity (rebalancing, governance, denom effect)
- Sells stake in 1 or N funds to a secondary buyer
- Price set by buyer's IRR target and discount-to-NAV
- Typical: 8-15% discount in normal, 30-50% in crisis
- Volume ~$70B/yr (2024) · cyclical
- GE-LAV: discount = $\pi(L,T\text{-}t)$ plus credit/quality adjustment

GP-led secondaries

- GP wants more time / capital for trophy assets
- Continuation fund vehicle (CV) created
- Existing LPs choose: cash out or roll
- Lazard/Evercore run the auction process
- Volume ~$80B/yr (2024) · structural growth
- GE-LAV: optimal exit boundary $L^*(t)$ — does CV beat exit?

Track 1 · Deliverable: secondary bid memo

Your task: as a secondary buyer at Ardian, bid on this LP-led portfolio.

Fund	NAV	Vintage	Strategy	Stage
Fund A	$25M	2018	US Buyout	85% deployed
Fund B	$15M	2020	Growth Eq	60% deployed
Fund C	$30M	2017	Infra	90% deployed
Fund D	$10M	2022	VC	40% deployed
Fund E	$20M	2019	Real Estate	75% deployed
TOTAL	$100M reported NAV	Mixed	Mixed	

Your bid should specify:

(1) Per-fund bid as % of NAV

(2) Aggregate portfolio bid

(3) Required IRR rationalizing each bid (use LA-IRR)

(4) Sensitivity: what if L drops 0.5 over 12 months?

Track 2 · MFG existence: Schauder setup

Goal: prove a mean-field game equilibrium exists for GE-LAV.

Strategy: cast equilibrium as a fixed point of a continuous map on a compact convex set, then apply Schauder.

Define the map T: $\mu \to \tilde{\mu}$ as:

Step 1	Given μ (proposed distribution of LP states), compute $\pi(L,T\text{-}t;\mu)$ — secondary market price
Step 2	Each LP solves HJB with this π, generating best-response policy $\alpha^*(L,t;\mu)$ and stopping rule $\tau^*(\mu)$
Step 3	Apply best-response policy to the OU dynamics, generating new cross-sectional distribution $\tilde{\mu}$
Step 4	Equilibrium $\Leftrightarrow$ fixed point $T(\mu) = \mu$

Track 2 · Verifying Schauder's hypotheses

Schauder requires: (i) compact convex space, (ii) continuous map, (iii) self-map.

✓ Compactness

Restrict μ to probability measures on a compact L-interval with bounded density. Use Prokhorov + tightness.

Lemma 3.1 in book

✓ Convexity

Set of admissible μ is convex by definition (mixtures of distributions are distributions).

Direct

✓ Continuity of T

π is continuous in μ in the Wasserstein metric. HJB solution V is continuous in π. Hence T continuous.

Lemma 3.2 in book

✓ Self-map

T(μ) is itself a probability measure on the L-interval — closed under T.

Direct

Schauder ⇒ a fixed point exists. QED (existence).

Track 2 · Uniqueness: monotonicity conditions

Existence is the easy part. Uniqueness requires more structure.

Lasry-Lions (2007) monotonicity condition:

$$\int (\pi(L;\mu_1) - \pi(L;\mu_2))\, d(\mu_1 - \mu_2)(L) \geq 0$$

Interpretation: a 'more distressed' distribution produces 'less favorable' prices — monotone aggregation.

When this holds (verified for OU + linear π in our calibration), the equilibrium is unique.

Theorem 3.4 (Asaf, 2026)

Under (A1)-(A4) of Section 3.2, the GE-LAV mean-field game admits a unique equilibrium μ^* with associated price π^* and policy α^*.

Track 2 · Problem set + readings

Due: end of Session 27

- **P1** Show: if μ has density bounded by M, then $T(\mu)$ has density bounded by $M' = M \cdot e^{(\kappa T)}$
- **P2** Verify continuity of T in the 1-Wasserstein metric (use Kantorovich duality)
- **P3** Show monotonicity (Lasry-Lions) holds for $\pi(L,T-t;\mu)$ linear in the mean of μ
- **P4** Counter-example: construct π non-linear in μ for which equilibrium is non-unique
- **P5** (Bonus) Extend uniqueness to the case where $r = r(L)$ is also state-dependent

Reading: Carmona-Delarue Ch. 2 (existence). Lasry-Lions (2007) JFA original paper.

Track 2 · Connection to Lasry-Lions seminal results

Where GE-LAV fits in the Mean-Field Game literature.

Lasry-Lions (2007) JFA

Original mean-field game framework; existence/uniqueness for general dynamics

Carmona-Delarue (2018)

Two-volume reference; FBSDE / master equation approach

Cardaliaguet et al. (2019)

Convergence of N-player games to the MFG limit

Where GE-LAV adds

First application to private capital pricing with secondary-market clearing

Open problem

Rate of convergence: how large does N (number of LPs) need to be for MFG to be accurate?

Both tracks reconvene: what we agree on

After today, Tracks 1 and 2 agree on:

Track 1 produced

An LA-PME calibration framework that can be applied to any LP-led secondary, plus a rigorous bid memo discipline.

Track 2 produced

An existence and uniqueness theorem for the GE-LAV mean-field equilibrium — verified under (A1)-(A4).

Common ground

The secondary market price $\pi(L,T-t;\mu)$ that Track 1 estimates empirically is the same object Track 2 proves exists uniquely.

Session 26 summary

What we accomplished today

1. VC has the largest Jensen bias by asset class — the 'PE alpha' debate is largely about this
2. Secondary market dynamics: LP-led vs GP-led each have ~$70-80B/yr volume
3. Track 2 proved MFG existence via Schauder fixed-point
4. Uniqueness follows from Lasry-Lions monotonicity — holds under calibrated GE-LAV

Next session

Session 27: Infrastructure case + Fokker-Planck equation derivation