

Private Credit Case / GE Equilibrium Existence

Session 28 · Ares Capital case · Market-clearing GE proof

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Primary Text: Liquidity Illusion (Forthcoming, 2026)

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What we'll cover today

1

Recap and track choice

2

Track 1: Direct lending fundamentals

What's different from PE

3

Track 1: Ares Capital case

BDC structure • GE-LAV applied

4

Track 1 deliverable

Allocation rec for pension fund

5

Track 2: GE-LAV market clearing

Walrasian setup

6

Track 2: Existence proof

Brouwer-Schauder argument

SAMIR ASAF

Recap: Session 27 • Infrastructure / Fokker-Planck

Three takeaways carried forward:

1

Long-horizon infra is where DCF errors are largest in absolute dollars (~\$0.5B+ per asset)

2

Fokker-Planck equation governs the evolution of the LP-state distribution

3

Master equation couples V (HJB) and μ (FP) — together define GE-LAV equilibrium

Today: private credit (T1) • GE existence (T2)

Today the class divides. Choose your seat:

TRACK 1 — Practitioner

Direct lending is shorter-duration than PE — but state-dependent default risk matters. Apply GE-LAV to a BDC.

TRACK 2 — Researcher

Prove that a general equilibrium (Walrasian sense) exists for GE-LAV market clearing. Different proof than MFG existence.

Reminder: Private credit is now \$1.7T globally — second-largest private asset class after PE.

Track 1 • How private credit differs from PE

Private credit

- ▶ Shorter horizon (3-5 yrs typical)
- ▶ Lower Jensen bias (low T^2 scaling)
- ▶ BUT: default risk is state-dependent
- ▶ Recovery rates collapse in stress
- ▶ Floating rate (rate exposure is the offset)
- ▶ Senior secured collateral

Private equity

- ▶ Long horizon (7-10 yrs)
- ▶ Large Jensen bias
- ▶ Default risk modest, but timing risk huge
- ▶ Recovery via secondary sale
- ▶ Equity (no senior claim)
- ▶ Cash-flow-based valuation

Track 1 • Ares Capital BDC case

Ares is the largest BDC by AUM (\$24B). A public proxy with quarterly transparency.

Metric	Pre-2022	2022-23 (rates)	2024	Comment
AUM (BDC)	\$18B	\$22B	\$24B	Grew through cycle
Total invested	\$22B	\$26B	\$28B	
NIM	8.4%	11.2%	10.8%	Floating rate benefit
Default rate	0.9%	3.5%	2.8%	State-dependent
Discount to NAV (mkt)	-4%	-18%	-9%	Cycle-dependent
Implied L from discount	+0.2	-0.8	-0.1	GE-LAV inference
GE-LAV π implied	2.5%	6.4%	3.1%	

Even short-horizon credit shows GE-LAV-consistent state-dependent pricing.

Track 1 • Deliverable: pension fund allocation rec

Setup

Mid-sized pension. \$50B AUM. Current allocation: 65 / 25 / 10 (eq/fixed/alt)

Question

Allocate \$2B to private credit. Direct lending fund A vs BDC fund B vs fund-of-funds C

Methodology

Apply LA-IRR + LA-PME framework. Use $\pi(L,T)$ for state-dependent yield

Output

1-page IC recommendation memo • 1 sensitivity table

Grading rubric

Numerics 40% • Reasoning 30% • Writing 20% • Sensitivity 10%

Track 2 • Walrasian market clearing for GE-LAV

Different from MFG existence: this proves a general equilibrium of supply and demand for liquidity.

Setup:

Agents	Continuum of LPs (sellers) and continuum of secondary buyers (demand)
Commodity	PE positions, indexed by (NAV, vintage, asset class)
Price	$p(L, T-t; \mu)$ — the secondary market clearing price (% of NAV)
Excess demand	$Z(p) = \text{demand}(p) - \text{supply}(p)$
Equilibrium	p^* such that $Z(p^*) = 0$ — market clears

We prove: p^ exists for every L . Hence $\pi(L, T-t; \mu)$ is well-defined.*

Track 2 • Existence proof outline

✓ **Step 1: Z continuous in p**

By continuity of buyer/seller optimization in price; standard envelope theorem.

✓ **Step 2: Z homogeneous of degree 0**

Walrasian property; no money illusion.

✓ **Step 3: Walras' Law**

$\sum p_i \cdot Z_i(p) = 0$ (budget constraints sum).

✓ **Step 4: Boundary behavior**

As $p \rightarrow 0$, $Z(p) > 0$ (demand exceeds supply); as $p \rightarrow 1$, $Z(p) < 0$. By IVT, equilibrium exists.

✓ **Step 5: Brouwer fixed-point alternative**

If multi-commodity, use Brouwer on the price simplex. Each map $T(p) \rightarrow p$ that adjusts toward $Z=0$ has a fixed point.

Theorem 4.1: For every L and μ , a clearing price $p^*(L, \mu)$ exists. ■

Track 2 • Problem set (due Session 29)

P1

Verify Walras' Law for GE-LAV: buyer and seller utility maximization implies $\sum p \cdot Z = 0$

P2

Construct a counterexample where Z is not continuous; show how the proof fails

P3

Show: existence does NOT imply uniqueness. Find conditions under which p^* is unique

P4

Connect this proof (market-clearing) to the MFG existence (Session 26). When are they equivalent?

P5

(Bonus) Compute equilibrium for a simple two-state economy (L =high vs L =low) by hand

Track 1 • BDC dividend mechanics

BDCs must distribute $\geq 90\%$ of taxable income. Why this matters for GE-LAV pricing.

Distribution requirement

RIC tax status: $\geq 90\%$ taxable income paid out

Implication

No deferral — every dollar earned shows up as cash

GE-LAV implication

Cash flow timing is largely fixed; less optimal exit flexibility

Market reflection

BDC market caps trade near NAV in steady state, but L-state sensitive

Calibration

Ares 2022-23 discount = -18% implies $L \approx -0.8$ (book Table 5.3)

Sanity check

Cross-reference with credit spread widening over the same period

Both tracks reconvene: what we agree on

After today, Tracks 1 and 2 agree on:

Track 1 produced

A pension allocation rec for \$2B into private credit, explicitly accounting for L-dependent default risk and short-horizon term structure.

Track 2 produced

A formal existence theorem for the GE-LAV market-clearing price $p^*(L, \mu)$ — guaranteeing the framework is well-posed.

Common ground

Private credit's 2022-23 discount widening (NAV-to-mkt 4%→18%) is what the existence theorem says must happen when L shifts — predicted, not surprising.

Track 1 • Distressed direct-lending playbook

When portfolio companies default, what does GE-LAV say to do?

Default identification

Coverage ratio breach • interest payment miss

Workout options

(1) Amend and extend (2) Equity-for-debt swap (3) Foreclose & sell

GE-LAV decision rule

Continue if $V(L,t) > \text{recovery value} \times (1 - \pi(L,T-t))$

Recovery rate (history)

Senior secured: 65-75% normal • 35-45% crisis

Workout horizon

12-24 months typical • drives the relevant T in $\pi(L,T)$

Pension fund question

Is the BDC's workout team strong? Track record matters more than NIM

Session 28 summary

What we accomplished today

- 1 Private credit has lower Jensen bias than PE but state-dependent default risk dominates
- 2 Ares Capital BDC case shows GE-LAV-consistent state-dependent pricing in real data
- 3 Track 2 proved general-equilibrium existence: a clearing price $p^*(L, \mu)$ exists for every (L, μ)
- 4 Combined with MFG existence (S26), GE-LAV is fully well-posed

Next session

Session 29: Digital & Climate / Jensen + Pigouvian Proofs

Track 2 • Connecting GE existence to MFG existence

Two existence proofs for two different objects. When are they equivalent?

MFG existence (S26)

Schauder on (μ, V, π) — value function fixed-point

GE existence (today)

Brouwer/Walrasian on p — clearing-price fixed-point

Are they the same?

Not in general. MFG \subset GE only when secondary market clears continuously

When MFG \Leftrightarrow GE

Linear demand curves, no rationing, full information

Counter-case

If secondary market freezes (2008), MFG holds but GE doesn't

Open research

Conditions for MFG-without-clearing — partial equilibrium concept