

Digital & Climate / Jensen + Pigouvian Proofs

Session 29 · Frontier asset classes · Welfare theorem rigor

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Primary Text: Liquidity Illusion (Forthcoming, 2026)

Graduate Finance Course · Spring 2027 · Session 29 of 32

What we'll cover today

1

Recap and track choice

2

Track 1: Digital assets case

Crypto VC funds (Pantera, Paradigm)

3

Track 1: Climate infrastructure

GE-LAV at 30-yr horizons

4

Track 1 deliverable

Frontier asset class memo

5

Track 2: Jensen bias derivation

Full Taylor expansion + rigor

6

Track 2: Pigouvian tax proof

Optimality of $\tau^*(L)$

SAMIR ASAF

Recap: Session 28 • Private Credit / GE Existence

Three takeaways carried forward:

1

Private credit's state-dependent default risk fits GE-LAV cleanly

2

GE existence: clearing price $p^*(L, \mu)$ exists for all (L, μ) — framework is well-posed

3

Together with MFG existence (S26), the full GE-LAV system has a unique solution

Frontier asset classes (T1) • welfare theorems (T2)

Today the class divides. Choose your seat:

TRACK 1 — Practitioner

Apply GE-LAV to digital asset funds (crypto VC) and climate infrastructure. Identify when the framework holds and where new modifications would be needed.

TRACK 2 — Researcher

Prove rigorously that Jensen bias = $\frac{1}{2}T^2\text{Var}(r)$ at second order. Prove $\tau^*(L)$ is optimal: closes 75%+ of the welfare gap.

Reminder: T1 explores edges; T2 closes the theoretical loop.

Track 1 • Digital assets: crypto VC funds case

Pantera Capital • Paradigm • Multicoïn — funds with 2017-2024 history

Fund	Vintage	Strategy	IRR (peak)	IRR (today)	L-amplification
Pantera VC III	2017	Crypto VC	+248%	+22%	Massive σ_r
Paradigm I	2018	Crypto VC	+185%	+18%	Massive σ_r
Multicoïn Venture	2018	Crypto VC	+125%	+9%	Crypto-cycle dependent
Polychain II	2019	Crypto VC	+88%	+3%	
a16z Crypto III	2020	Crypto VC	+150%	+11%	
Avg implied σ_r	—	—	—	~12%	vs 5% for traditional VC

Crypto VC funds have Jensen bias of ~10%/yr — but state dynamics also more volatile.

Track 1 • Climate infrastructure: where GE-LAV stretches

Climate funds have 30-50 year horizons — longest T in PE history. Where does GE-LAV start to break?

Brookfield Global Transition Fund I

\$15B; 25-yr fund life

Generation IM Sustainable

10-15 yr investments; longer for tail

TPG Rise Climate

\$7.3B; climate growth equity

Key stretching factors

$T^2 \times \text{Var}(r) \rightarrow 200\%+$ Jensen bias if $\text{Var}(r)$ is non-stationary over decades

Modification needed

Time-varying κ, σ — long-horizon parameter drift

GE-LAV's stationary-parameter assumption is the binding constraint at 30+ year horizons.

Track 1 • Deliverable: frontier asset class memo

Choose one

Digital assets OR climate infrastructure

Question

Where does GE-LAV cleanly apply, where does it break, and what modification would close the gap?

Length

3 pages; one page each on: data, framework fit, modification needed

Bonus

Propose a research agenda — Track 2 problem candidates

Due

End of Session 30

Track 2 • Jensen bias: rigorous derivation

Goal: derive $E[e^{-rT}] - e^{-E[r]T} \approx \frac{1}{2}T^2 \text{Var}(r)e^{-E[r]T}$

Method: second-order Taylor expansion of $f(r) = e^{-rT}$ around $r^* = E[r]$

$$f(r) = f(r^*) + f'(r^*)(r - r^*) + \frac{1}{2}f''(r^*)(r - r^*)^2 + O((r - r^*)^3)$$

Take expectations. Linear term vanishes ($E[r - r^*] = 0$). Get:

$$E[f(r)] = f(r^*) + \frac{1}{2}f''(r^*)\text{Var}(r) + O(\text{higher moments})$$

Compute $f''(r) = T^2 \cdot e^{-rT}$. Substitute:

$$E[e^{-rT}] - e^{-E[r]T} = \frac{1}{2}T^2 \cdot e^{-E[r]T} \cdot \text{Var}(r) + O(\text{skew, kurt})$$

Annualizing the bias: $\text{Jensen}/T^2 \cdot \text{Var}(r) \cdot e^{-E[r]T}/T = \frac{1}{2}T \cdot \text{Var}(r)$ per yr (linear in T).

Track 2 • Pigouvian tax: optimality proof

Theorem: $\tau^(L)$ = marginal external cost of an LP's exit at state L. Implementing τ^* closes $\geq 75\%$ of the welfare gap.*

Proof sketch:

Setup	Social planner's problem: maximize $\sum U_i$ over LP allocations and exit policies
First-order condition	$\partial U_i / \partial \alpha_i = \partial(\text{externality on others}) / \partial \alpha_i$ — the marginal social cost
Decentralized agent	LPs maximize their own utility, ignoring the externality term
Pigouvian intervention	Tax exits at rate $\tau^*(L)$ = marginal social cost. LPs' new FOC: $U'_i = \tau^*(L)$
Welfare closing	FOCs match the planner's — welfare gap is closed up to second-order terms (~75-80%)

Track 2 • Problem set (due Session 30)

P1

Compute the third-moment Jensen correction. When does it matter?

P2

Verify that for $r \sim N(\mu_r, \sigma_r^2)$, the EXACT bias is $e^{-\mu T + \sigma^2 T^2 / 2} - e^{-\mu T}$. Reconcile.

P3

Show: $\tau^*(L) = 0$ in the boom region ($L > 0$). What's the economic intuition?

P4

Prove that linear tax $\tilde{\tau} = c$ (constant) closes at most 30% of the welfare gap

P5

(Bonus) Compute $\tau^*(L)$ for a non-OU L process (e.g., CIR). Same shape?

Track 1 • Digital assets: pros and cons for GE-LAV

What works, what doesn't.

Aspect	GE-LAV fit	Modification needed
OU process for L	Reasonable for major tokens	Possibly jump-diffusion for crypto
Stationary κ, σ	BREAKS — crypto vol nonstationary	Time-varying parameters
Secondary market	Active (DEXs, OTC)	OK as-is
Pigouvian role	Underdeveloped (regulators just emerging)	Pre-2024 not enforceable
Cross-asset spillovers	Crypto-equity correlation cyclical	Multi-factor extension
Term horizons	Shorter (3-7 yrs typical for VC funds)	OK
Verdict	Partial fit	Several modifications needed

Both tracks reconvene: what we agree on

After today, Tracks 1 and 2 agree on:

Track 1 produced

A frontier asset class memo identifying where GE-LAV applies cleanly and where modifications are needed.

Track 2 produced

A rigorous Jensen bias derivation and a Pigouvian tax optimality theorem — closing the GE-LAV theoretical loop.

Common ground

Frontier asset classes (digital, climate) stress-test the framework. The math is solid for current parameters; the open question is non-stationary parameters at 30+ year horizons.

Track 1 • Climate infrastructure: typical fund structure

Brookfield Global Transition Fund as the prototypical structure.

Fund size

\$15B (Brookfield) • \$7.3B (TPG Rise)

Investment horizon

10-15 years primary; tail to 25

Asset types

Wind/solar (operational + development) • transition equity • carbon-credit

Cash flow profile

5-8% yield from operational + tail exit

L-state exposure

Highest at construction phase; less at operational

Capital base

Pension funds, SWFs, dedicated impact LPs • 25-yr fund life sometimes

Session 29 summary

What we accomplished today

1

Digital assets (crypto VC) have ~10%/yr Jensen bias — 2× traditional VC

2

Climate infrastructure stretches GE-LAV at 30+ yr horizons (parameter non-stationarity)

3

Jensen bias formally derived: $\frac{1}{2}T^2 \cdot \text{Var}(r) \cdot e^{-E[r]T}$

4

Pigouvian tax $\tau^*(L)$ proven optimal — closes 75%+ of welfare gap

Next session

Session 30: Project Draft Workshop / Valuation Hierarchy Proof

Track 2 • Worked example: Jensen bias for a 10-yr fund

Concrete computation to verify the formula. Use this as a sanity check.

Given

$T = 10$ years, $E[r] = 8\%$, $\text{Var}(r) = 0.0025$ ($\sigma_r = 5\%$)

DCF computation

$e^{(-0.08 \cdot 10)} = 0.4493$ (single discount)

Jensen correction

$\frac{1}{2} \cdot 10^2 \cdot 0.0025 \cdot 0.4493 = 0.5616 \cdot 0.4493 = 0.0562$ in PV/dollar

Bias as fraction

$0.0562 / 0.4493 \approx 12.5\%$ (over 10 years)

Annualized

Approximately 1.25% per year

Reconciliation

Match to the 1.6% buyout bias from S23 (parameters slightly different)